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Sign problem in Z-coefficient for particle emission angular distributions

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There are two definitions of Blatt-Biedenhan's Z-coefficient, the one in the original paper [1]

$$Z(l_1 j_1 l_2 j_2; sL) = i^{-l_1+l_2+L} \hat{l}_1 \hat{l}_2 \hat{j}_1 \hat{j}_2 \langle l_1 l_2 00 | L0 \rangle W(l_1 j_1 l_2 j_2; sL), \tag{1}$$

and another one by Lane and Thomas [2]

$$\overline{Z}(l_1 j_1 l_2 j_2; sL) = \hat{l}_1 \hat{l}_2 \hat{j}_1 \hat{j}_2 \langle l_1 l_2 00 | L0 \rangle W(l_1 j_1 l_2 j_2; sL), \tag{2}$$

where the difference is the phase factor $i^{-l_1+l_2+L}$. We follow the notation by Lane and Thomas.

The angular distribution of shape elastic scattering (for neutrons) is given by the scattering amplitudes $A(\theta)$ and $B(\theta)$ at a given center-of-mass angle θ;

$$A(\theta) = \frac{i}{2k} \sum_l \left\{ (l+1)(1 - S_{l,l+1/2}) + l(1 - S_{l,l-1/2}) \right\} P_l(\cos\theta), \tag{3}$$

$$B(\theta) = \frac{1}{2k} \sum_l \left\{ S_{l,l-1/2} - S_{l,l+1/2} \right\} P_l^1(\cos\theta), \tag{4}$$

$$\left(\frac{d\sigma}{d\Omega} \right)^{\rm SE} = |A(\theta)|^2 + |B(\theta)|^2, \tag{5}$$

where S_{lj} is the optical model S-matrix element, $P_l(\cos\theta)$ is the Legendre function, and $P_l^1(\cos\theta)$ is the associated Legendre function. The same angular distribution can be expanded by the Legendre function as

$$\left(\frac{d\sigma}{d\Omega} \right)^{\rm SE} = \sum_L B_L P_L(\cos\theta), \tag{6}$$

and the coefficients are given as

$$B_L = \frac{1}{8k^2} \sum_{l_1 j_1 l_2 j_2} \left[\overline{Z}(l_1 j_1 l_2 j_2; \frac{1}{2} L) \right]^2 \Re \left\{ (1 - S_{l_1 j_1})(1 - S_{l_2 j_2})^* \right\}. \tag{7}$$

This expression is independent of the target spin I_A. Since the $\overline{Z}$-coefficient is squared, the phase facotr of $Z(l_1 j_1 l_2 j_2; 1/2 L)$ does not matter. In this expansion, $4\pi B_0$ is equal to the angle-integrated elastic scattering cross section. To confirm the equality of Eq. (5) and Eq. (6) we calculated the shape elastic scattering angular distribution for ^{152}Gd at 20 MeV, with both equations, shown in Fig. 1. Koning-Delaroche global optical potential was used.

The differential cross section for the compound reaction is written in the same way as the shape elastic scattering;

$$\left(\frac{d\sigma}{d\Omega} \right)_{ab} = \sum_L B_L P_L(\cos\theta_b), \tag{8}$$

in which a particle a with the spin i_a collides with the targe nucleus A with the spin I_A, forming a compound state J, then decays into a channel where a particle b with the spin i_b is emitted leaving the residual nucleus B with the spin I_B. Parity is not written explicitly, but it must be conserved at each spin coupling.

The B_L coefficient is given by Moldauer's statistical theory as

$$B_L = \frac{1}{4k^2} \frac{(-)^{I_B - I_A + i_b - i_a}}{(2i_a+1)(2I_A+1)} \frac{1}{N} \sum_J (2J+1)^2 \sum_{l_a j_a} \sum_{l_b j_b} W_{ab} \left\{ X_{l_a j_a}(E_a) X_{l_b j_b}(E_b) + \delta_{I_A I_B} \delta_{E_a E_b} Y_{l_a j_a, l_b j_b}(E_a, E_b) \right\}, \tag{9}$$

where W_{ab} is the width fluctuation factor, and

$$X_{lj}(E) = \overline{Z}(ljlj; iL) W(jJjJ; IL) T_{lj}(E), \tag{10}$$

$$Y_{l_a j_a, l_b j_b}(E_a, E_b) = (1 - \delta_{l_a l_b})(1 - \delta_{j_a j_b}) \left\{ \overline{Z}(l_a j_a l_b j_b; i_a L) W(J j_a J j_b; I_A L) \right\}^2 T_{l_a j_a}(E_a) T_{l_b j_b}(E_b) \tag{11}$$

and the normalization N is given by integrating/summing all possible decay channels.

$$N = \sum \int T_{lj}(E) dE. \tag{12}$$

For the Hauser-Feshbach theory, $W_{ab} = 1$ and $Y_{l_a j_a, l_b j_b}(E_a, E_b) = 0$.

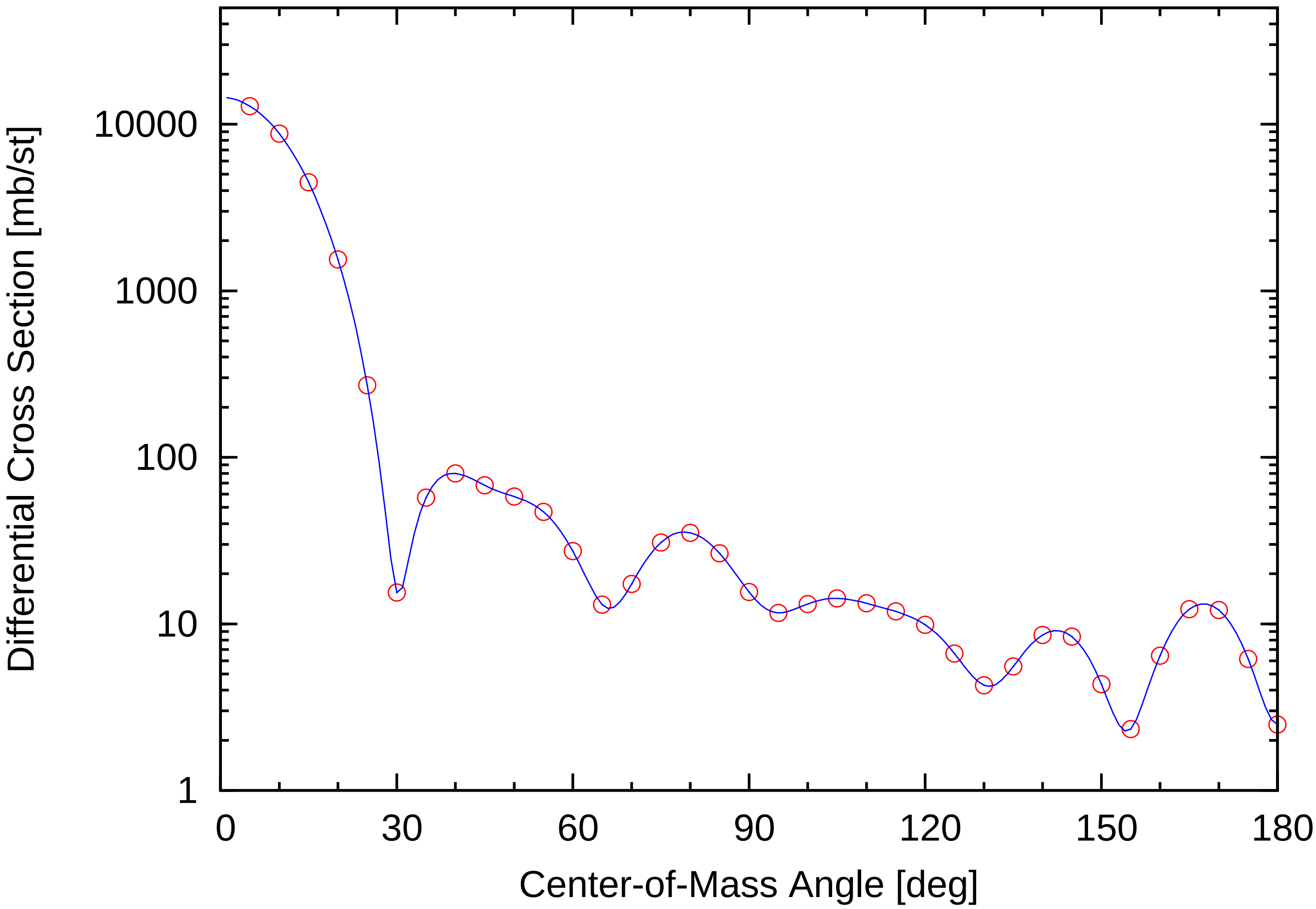

FIG. 1. Calculated shape elastic scattering angular distribution of ^{152}Gd at 20 MeV. The symbols are from the scattering amplitudes at each angle, and the curve is from the Legendre coefficients.

The term that includes $X_{l_a j_a}(E_a) X_{l_b j_b}(E_b)$ has the factor of $\overline{Z}(l_a j_a l_a j_a; i_a L)\overline{Z}(l_b j_b l_b j_b; i_b L)$. When the Z-coefficients are given by the original definition,

$$
\begin{aligned}
Z(l_a j_a l_a j_a; i_a L) Z(l_b j_b l_b j_b; i_b L) &= i^{2L}(2l_a+1)(2j_a+1)(2l_b+1)(2j_b+1) \\
&\times \langle l_a l_a 00|L0\rangle \langle l_b l_b 00|L0\rangle \\
&\times W(l_a j_a l_a j_a; i_a L) W(l_b j_b l_b j_b; i_b L).
\end{aligned} \tag{13}
$$

Since L is always even for the compound reaction, i^{2L} becomes just one. Therefore the Legendre coefficient B_L will be identical for the both definition, Z and $\overline{Z}$ (note that Z-coefficient in Y is squared). As an example, the neutron elastic scattering angular distributions for 2-MeV neutron induced reaction on ^{58}Ni are shown in Fig. 2, where the compound elastic scattering is 90-degree symmetric by definition.

[1] J.M. Blatt, L.C. Biedenharn, Rev Mod Phys. **24**, 258 (1952).
[2] A. M. Lane, R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

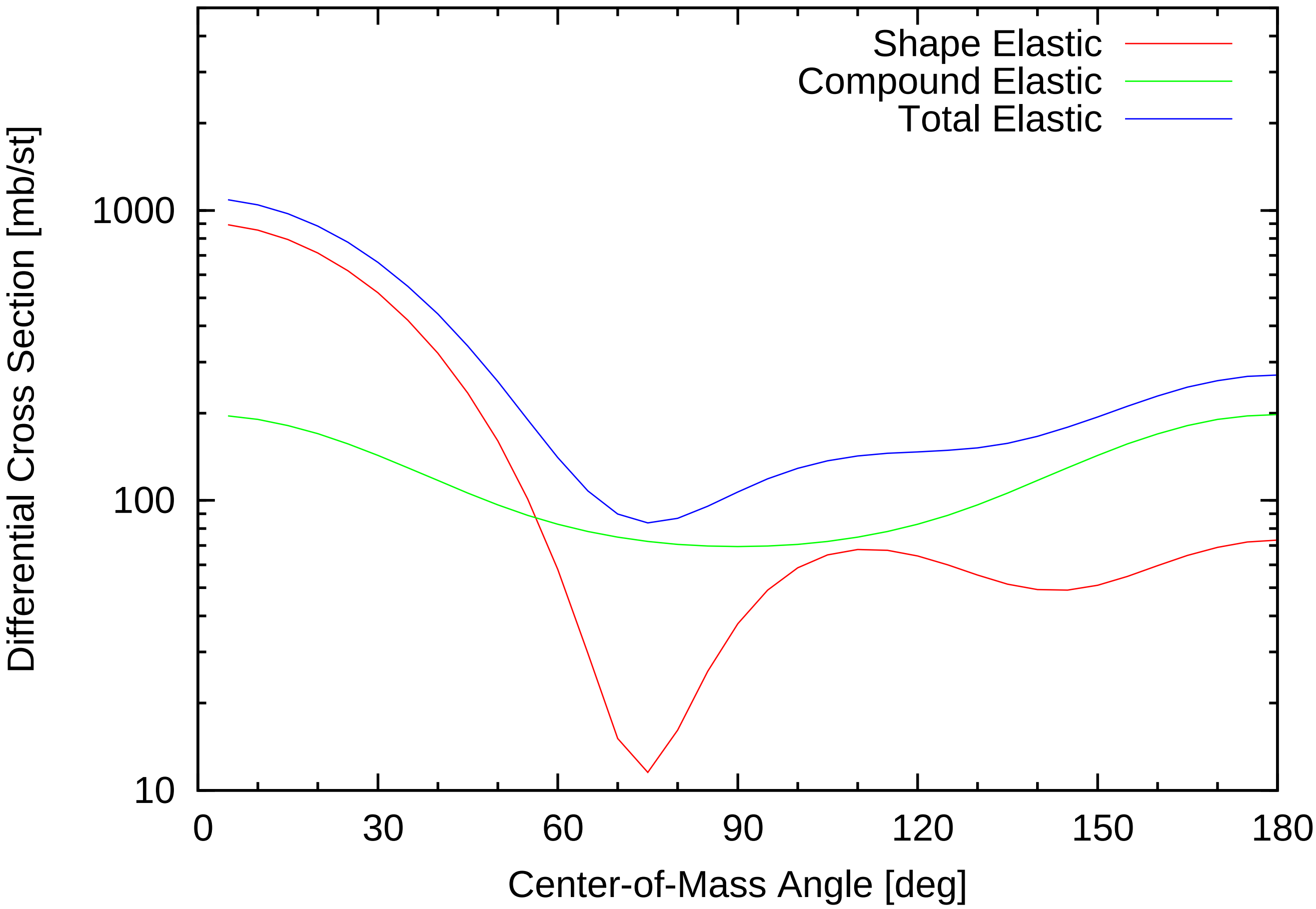

FIG. 2. Calculated shape and compound elastic scattering angular distributions for ^{58}Ni at 2 MeV.